Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 3, 1999

ADEQUACY OF THE DESCRIPTION OF THE PROCESS OF FRACTURE IN COMPUTER SIMULATIONS

A. G. Ivanov

UDC 533.06.01

A number of studies devoted to the description of the entry of a celestial body into a planetary atmosphere are analyzed. It is shown that one can obtain an adequate qualitative pattern of the phenomenon with the use of an integral fracture criterion if the necessary energy condition of fracture is satisfied. The use of the traditional fracture criteria for materials does not give a correct description of the process considered.

In the middle of the twentieth century, the need to explain the reasons of catastrophic brittle fractures of bulky objects led to revision of the standard fracture criteria. Linear fracture mechanics (LFM) was developed based on the Griffith criterion of the transition of a brittle crack to an unstable state [1]. In LFM, fracture is the result of work executed by the elastic energy of deformation. The need arose to revise the fracture criteria and to elaborate new methods of testing the materials for strength. The role of the standard strength criteria, such as the critical values of the rupture stress σ_*, the shear stress τ_*, and the strain ε_* or their combinations, was restricted to a comparison of the materials under the standard conditions of a test. New fracture criteria, including the specific work of fracture per unit surface G_{Ic} used, as a rule, in the form $G_{Ic}E \sim K_{Ic}^2$ (E is Young's modulus and K_{Ic} is the critical value of the coefficient of stress intensity in the mouth of a crack), were called on. The methods of their determination were also developed. The standard criteria of materials strength corresponded to the singular points of the diagrams of material deformation and were easily found. In LFM, it is necessary to perform a series of tests on special and geometrically similar specimens of different size to obtain the quantities G_{Ic} or K_{Ic} [2]. In Cherepanov's opinion [3], "the strength of a structure is always a certain random quantity, because, first, the exact sites of all defects are not known beforehand, and, secondly, if the exact location of these sites were known, the solution of the corresponding mathematical problem would be impossible because of its complexity." Cherepanov also recommended a sequence of operations in strength computations [3, Appendix 1].

In some cases, the way out is to use the solutions of a series of simple problems in which one or several cuts of definite length oriented in a definite way relative to the specimen considered and the forces applied are considered instead of arbitrarily located cracks [4]. A large number of such problems (the problems of K-calibration) are cited in [3, Appendix 1]. It follows from analysis of these problems and experimental fracture studies of geometrically similar objects in the elastic region of deformation [5] that strong large-scale effects of energy nature (LSEEN) can occur. The physical nature of these effects consists in the fact that in going from a certain object to its geometrically similar analog of a larger size, the elastic strain energy increases as L^3, and the work executed during crack propagation upon fracture increases only as L^2 (L is the characteristic size of the geometrically similar object). The consequence of this transition is the decrease in the specific strength (the fracture stress) of the larger object. We shall illustrate it by referring to the fracture of a space body (SB) when it enters into a planetary atmosphere [6–11].

Understanding the complexity and, sometimes, the impossibility to apply LFM, some authors recommended to use the criteria of materials strength in computer simulations. Here, the LSEEN is not

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Institute of Experimental Physics, Sarov 607190. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 3, pp. 191–195, May–June, 1999. Original article submitted March 28, 1997; revision submitted July 28, 1997.

taken into account, which suggests that the results of these studies might be criticized.

The intense state of a spherical SB was calculated by Fadeenko [6] and Korobeinikov et al. [10]. In [6], the pressure P on the surface of the sphere was distributed according to the following Newtonian law:

$$P = \rho v^2 \cos^2 \varphi \quad \text{for} \quad 0 \leqslant \varphi \leqslant \pi/2; \qquad P = 0 \quad \text{for} \quad \pi/2 \leqslant \varphi \leqslant \pi.$$

Here ρ is the atmospheric density and v is the SB velocity, and φ is the angle between the normal to the surface of the sphere and the direction of its motion. As one should expect, the tensile stress σ is reached at the point opposite to the critical point, and its maximum value is $\sigma \sim 0.365\rho v^2$. The intensity of the tangent stresses T is maximum inside the sphere over the circumference $\varphi \sim 60°$ at a distance of 0.25–0.35 of the radius from the center and reaches $T \sim 0.265\rho v^2$.

In [10], a more exact approximation $P(\varphi)$ was used and more complete calculation results were obtained. These results agree with those given in [6]. As a local fracture criterion convenient for calculations, the limit of shear strength [6] or the equality of the tangent stresses to the strength limit upon tension [10] is usually used. Grigoryan [7–9] considers that as soon as the aerodynamic load reaches the critical value of the fracture stress ($\rho v^2 \sim \sigma_*$), a fragmentation wave starts to travel over the body and the SB will be completely fractured for $\tau \sim D/C$ (D is the diameter of the sphere and C is the volumetric velocity of sound), i.e., the material loses its viscosity. According to [8], the fractured mass becomes close, in properties, to a liquid. It spreads and is carried away by the flow. In [10], three diagrams illustrate the process of increasing a region inside the SB in which the specified fracture criterion holds. Assuming that the core of the Schoemaker-Levy comet is a sphere of diameter 800 m ($v = 65$ km/sec, the material is ice, and the angle of entry into Yupiter's atmosphere is $\varphi = 45°$), complete fracture occurs in the interval of heights from 459 to 426 km, i.e., for 0.7 sec, which corresponds to $\tau \sim D/C$. Similar calculations for fracture of the Tunguska SB and the Sikhote-Alin' meteorite are also given.

Kondaurov et al. [11] used the same fracture criteria as those in [6–10], but assumed that the SB is shaped like a cylinder. It was noted that at the stage of fracture "... disintegration of the material which, is induced by shear stresses, prevails." The authors of [11] explained this form of fracture by the absence of shock waves of sufficient intensity for fracture by shear cracks.

The further process of interaction between the SB's dispersed material and a planetary atmosphere is described depending on the type of entry of a cumulative jet into an obstacle [6–9, 11].

To what extent is the description of the stage of fragmentation, dispersion [7], loss of viscosity [9], complete fracture [10], and fragmentation [11] true in the studies considered? We believe that the SB consists of a nonporous material of density $\rho > 1$ g/cm^3 and contains, as any real body, a certain amount of structural defects. The low temperature (~100 K), the large sizes (~10 m or greater), and the high rate of loading of the SB during its entry into a planetary atmosphere (~10^{-1}–10^2 sec^{-1} or greater) suggests that the fracture is brittle and occurs in the elastic region of deformation.

We turn to the experiment. The loading of the SB during its entry into a planetary atmosphere has a quasistatic character. Under these loads, brittle bodies usually disintegrate into two parts,[1] and only in going to shock loading does the number of fragments increase with intensity of loading. If one takes into consideration the forces of inertia acting on the SB during its motion in a planetary atmosphere, the stress states of the SB and the brick are qualitatively close.

We turn out attention to some circumstances. The compression-induced fracture of a nonporous body occurs only in the case where there is a possibility to realize transverse positive strain of the body. As follows from the calculation results given in [10], this strain is absent when the SB fractures. However, the process of fragmentation of a nonporous SB cannot occur without increasing its volume owing to the voids formed between the fragments. Therefore, the satisfaction of the fracture condition in a certain region inside the SB which is specified in a calculation does not mean that the material in this region fractures.

It is noteworthy that the elastic energy spent to disperse the SB's material is so many times greater

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[1] This study does not concern specially prepared defectfree materials, in which one can store a large amount of elastic energy, and their fracture will be of explosive character with numerous fragments.

than that spent during the fragmentation of the SB into two parts as the surface of the formed particles is greater than the surface of one open crack intersecting the SB. This situation is energetically preferred and should occur. The difficulties of the direct use of LFM were mentioned above. In the case considered, where it is impossible to obtain data on the SB defectness, these difficulties are even greater. At the same time, it has been shown above that, departing from LFM and using the fracture criteria of materials resistance, we leave the possibility of occurrence of the LSEEN outside the scope of analysis. Therefore, it is expedient to turn to the integral approach in the fracture problem [5]; like LFM, this approach is based on the energy aspect of the phenomenon, and its use is justified in some cases.

Denoting the specific value of the elastic energy, the area of the surface of an open crack, and the volume of the object by q, S, and V, respectively, we formulate the no-fracture condition in the general case:

$$\int_V q\,dV < \int_S G_{Ic}\,dS. \tag{1}$$

The use of the integral approach allows one to simplify considerably the procedure of finding the solution, as done, for example, in the determination of the nature of catastrophic fractures of pipelines [12] and in the development of the concept of operational reliability of pipeline transport [13].

Changing the sign of the inequality in (1), we obtain the desired fracture condition. However, to satisfy this condition, there should be a defect of critical size at which the crack "starts." The size of this defect is closely connected with the excess elastic strain energy. It is natural that this information is not available. Obviously, as the SB enters into the atmosphere, the store of elastic energy in it increases, and fracture occurs at a certain moment. In many aspects, the situation is similar to spallation in the interaction between rarefaction waves, where the initial defectness of the material is not investigated beforehand. Nevertheless, in these experiments the quantity λ (the analog of G_{Ic}) is easily determined under the assumption that an equality sign is present in expression (1) [14]. The value of λ found in the experiments with organic and inorganic glasses and several kinds of steel is approximately 30 times greater than G_{Ic}. Evidently, the left-hand side of (1) should be 30 times greater than the right-hand part to fracture the SB. These actions are not quite correct and can lead to significant quantitative errors; however, we obtain a physically correct, qualitative pattern of SB fracture.

It is necessary to note that Yu. I. Fadeenko was the first to attempt to allow for the scale factor in [6], where a statistical theory of strength that does not describe the experiment [3] is used, and the hypothesis of equilibrium fragmentation is adopted.

Ivanov and Ryzhanskii [15, 16] considered the Tunguska phenomenon without including the thermal effects. Describing the phenomenon in accordance with LFM and the integral approach, they passed from the standard fracture criteria which are based on the specific magnitude of the force, to energy relations. Changing the inequality sign in (1) to an equality sign (with a view to using the effective value of G_{Ic}) and estimating q as

$$q = \frac{\sigma^2}{2E} = \frac{\rho^2 v^4}{2E},$$

we obtain the fracture condition for the SB in the form of a sphere of diameter D_0:

$$\rho^2 v^4 = 3K_{Ic}^2/D_0. \tag{2}$$

The formed fragments again are considered as spheres of diameter $D_1 = 2^{-1/3}D_0$. With increase in the aerodynamic drag and with D_0 replaced by D_1, Eq. (2) is satisfied. Subsequent division of the formed fragments occur, and so on, until the aerodynamic drag stops to increase.

The computations have shown that:

— The fragmentation of the SB during its entry into the atmosphere is of multistage character owing to the LSEEN;

— The loading of the fragments between the moments of fracture is purely quasistatic;

— When the process of SB disintegration stops, the characteristic size of a fragment is determined by the velocity of motion of the fragment and the strength characteristics of the material.

Using the stage-by-stage process of SB fragmentation, one can describe the process of formation of a chain of fragments of the Schoemaker–Levy comet and to explain the possible way of formation of tektites and to establish the nature of numerous craters (about 100) formed by the Sikhote-Alin' meteorite. In the latter case, there is no need to assume that the meteorite was a body "glued " together from separate pieces of the SB, as done in [10]. This phenomenon is of interest also due to the fact that the information on the number of craters and the SB material allows one to estimate more correctly the effective G_{Ic}.

In our opinion, the examples of the interaction between the SB and a planetary atmosphere have highlighted the problem of the choice of fracture criteria for computer simulations. Application of the standard fracture criteria, established as early as Galilei's times, which are based on the critical values of the stress, the strain, etc., does not allow one to describe adequately the fracture process even in the simplest linear strain and fracture region. Evidently, the use of other criteria of local fracture in computer programs should be tested in experiment-simulating test problems of fracture. In particular, in test LFM problems, computations for geometrically similar objects of different scale should be performed with account of the LSEEN.

The author expresses his gratitude to Academician Yu. B. Khariton (deceased) for his interest in this work.

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